Stability properties of delay tolerant networks with buffered relay nodes

Vineeth Bala Sukumaran and Chandramani Singh
{vineethbs@iist.ac.in, chandra@dese.iisc.ernet.in}

***Abstract*—We study stability properties of single-source single-destination delay tolerant networks with random packet arrivals and buffered relay nodes, using source spray and wait routing. We derive the stability threshold, the supremum of arrival rates for which the source queue is stable, as a function of the buffer space at the relays. In particular, we show that the stability threshold only doubles as the relays' buffer size increases from one to infinity for a network without packet delivery feedback. For the system without packet delivery feedback, we propose lower bounds for the average queueing delay and average delivery delay for packets and compare with simulations. We also obtain the stability threshold numerically for a network with instantaneous packet delivery feedback.**

I. Introduction

Delay tolerant networks (DTNs) are comprised of nodes connected by links which have intermittent connectivity. In order to transport packets reliably in such networks, the store and copy paradigm is used in DTN routing and scheduling protocols such as Epidemic [1], Spray and Wait [2], Spray and Focus [3], Maxprop [4], Rapid [5], and Prophet [6]. Such DTN routing protocols copy packets from the source to multiple *relay nodes* so that there is diversity in the packets' path to the destination. This leads to an improved probability of packet delivery within a target time and average delivery delay. However, copying of packets from a source to a relay node requires the relay to have enough storage or buffer space to store the packet copies until it can be delivered or copied again. An important engineering problem is provisioning the storage or buffer space at the nodes, which can be used to tradeoff various quality of service metrics of the network.

In this paper, we consider the effect of storage or buffer size of the relays on performance metrics such as queue stability region, average queueing delay, and average delivery delay for a DTN with a single source and destination of packets. The DTN uses the source Spray and Wait (SW) routing protocol [2] to copy packets to mobile relay nodes which have finite buffer capacity to hold the packets. We obtain conditions for stability of the source queue, an analytical lower bound for the average queueing delay experienced by packets at the source queue, and a lower bound for the average delivery delay of packets.

The first author is with the Department of Avionics, Indian Institute of Space Science and Technology, Thiruvananthapuram, Kerala. The second author is with the Department of Electronic Systems Engineering, Indian Institute of Science, Bangalore. The authors would like to thank Prof. Rajesh Sundaresan (Dept. of Electrical Communication Engg., IISc) for useful discussions.

We assume that the source generates packets at the epochs of a Poisson point process. In contrast to prior work on DTNs which have assumed *lone packet* models (only a single packet exists within the network at a time), we consider queueing stability and queueing delay for DTNs with random arrivals as a function of the buffer size at relay nodes. We consider Source Spray and Wait (SW) protocol [1] which copies a single packet K times to relays. The SW protocol consists of a spray phase and a wait phase. In the spray phase the source copies the packet to the first K relays that it meets. But a relay that is carrying the packet cannot copy it to another relay. After the source copies the packet to K relays, the protocol enters the wait phase which continues until the packet reaches the destination. In our prior work [7] we had studied stability and delay performance for a similar DTN model but with extremely resource limited relay nodes which have a unit buffer capacity. In this paper, we consider stability and delay performance when all relay nodes have a buffer size of B. We obtain that the stability region of the DTN at most doubles when the buffer size B increases to infinity from one.

Lee et al. [8] study the asymptotic scaling of stability region and delay in a DTN as the number of nodes N increases. We note that this paper provides non-asymptotic results compared to the scaling laws which were obtained in [8]. Also, [8] assumes that all relays share a common buffer, while we assume per-relay buffers which are more realistic. Herdtner et al. [9] consider the effect of finite buffers on the sum throughput of a mobile adhoc network and shows that the sum throughput degrades substantially compared to systems with infinite buffers. Mahendran et al. [10] propose an asymptotic large-deviations based scheme to provision relay buffer sizes in a DTN so that performance is close to a DTN with infinite relay buffer size. Using our stability results we also show that a buffer size of 19 is enough to achieve 95% of the maximum achievable throughput.

We note that prior work on DTNs with packet arrivals (e.g., see [2], [4], [5], [6]) have used simulation tools (such as ONE [11]) to understand the effect of protocol parameters on quality of service for packets. Ramaiyan et al. [12] considered a vehicular DTN with $K = 1$, in which packet need not be copied to the first relay. They study the problem of discovering the optimal relay on the go. Groenevelt et al. [13] modeled epidemic relaying and two-hop relaying using Markov chains. They derived the average delay and the number of copies generated until the time of delivery. Zhang et al. [14] developed a unified framework based on ordinary differential equations (ODEs) to study epidemic routing and its variants. Altman et al. [15] addressed the optimal relaying problem for a class of *monotone relay strategies* which includes epidemic relaying and two-hop relaying. Singh et al. [16] considered the tradeoff between delivery delay and number of copies in a DTN where a single packet has to be transmitted to multiple destinations. However, in this paper we analyze the case where multiple packets need to be transmitted to a single destination.

Outline and contributions: We present the DTN model in Section II. We also state the definitions of the stability region and the delay metrics in this section. We obtain the stability region (threshold in the case of single source-destination) for the DTN without packet delivery feedback in Section III. The characterization of the stability threshold as a function of DTN parameters and especially the buffer size is our primary contribution in this paper. We also discuss the similarities and differences between our results and [8] in this section. We also obtain the stability region for a DTN with unit buffer size but with feedback of packet delivery information in Section IV; this is a secondary contribution. We then consider the delay performance of the DTN without packet delivery feedback in Section V. We obtain an analytical lower bound on the average queueing delay of packets as well as a lower bound on the average delivery delay which form secondary contributions. Simulations of queueing delay and delivery delay in the DTN and their comparisons with our obtained bounds are also presented in Section V.

II. System model and problem statement

We consider a continuous time mobile ad-hoc network model (see Figure 1) similar to that in [16] and [7]. Our presentation of the model is similar to that in our prior work [7]. The time index is denoted as t. We assume that the network consists of $N+2$ mobile nodes, comprising of a source node, a destination node, and N relay nodes. The source is assumed to have fixed length packets arriving to it at the random points of a Poisson point process of rate λ. The packets have to be transferred to the destination and are assumed to be queued in an infinite length buffer at the source. The source transfers the data packets to the destination through the mobile relay nodes using two-hop relaying. We assume that the source does not meet the destination directly*. We say that a relay has met the source/destination when the relay node comes within the communication range of the source/destination.

For improving the delivery delay, we assume that the source copies each packet to at most K separate relays. We assume that a relay has a finite buffer which can hold at most $B \in \mathbb{Z}_+$ packets. When a source meets a relay, the source finds out whether the relay has a free buffer space or not. If the relay has a free buffer space, then the source finds out the list of packet copies that the relay is not currently carrying but which are there in the source queue and copies one copy of such a packet to the relay†. A relay with a free buffer space is said to be susceptible, while a relay which has a packet copy is said to be infected by that packet.

A packet is delivered when any of relays infected with that packet first meets the destination and copies the packet to the destination. We consider two cases: (a) *no-feedback* - we assume that there is no feedback mechanism employed by the destination by which the source and the infected relays would get information about packet delivery, and (b) *instant-feedback* - we assume that there is instantaneous feedback about packet delivery from the destination to the source and relay. In the no-feedback case, we assume that a packet is removed from the source buffer just after K copies have been made and the relay buffer space is freed after the relay copies that packet's copy to the destination. In the instant-feedback case, we assume that a packet is removed from the source and relay buffers when it is first delivered to the destination.

We assume that the intermeeting time processes of relay nodes with the source and destination nodes are independent, with each intermeeting time distributed as an exponential random variable with rate β (an assumption motivated by [13]). For the r^{th} relay the buffer size at time t is denoted as $B^r(t)$.

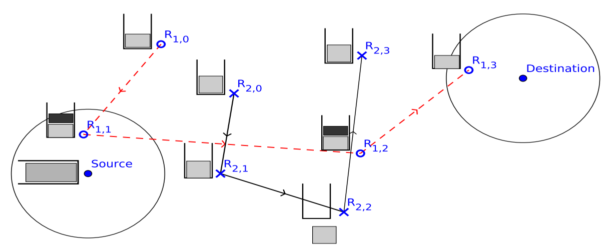

Fig. 1: An example network with two relays. The relays store packets in the buffers as shown. The source and destination communication ranges are shown as circles centered at the source and destination. The relative position of relay R_i at four different times is shown as $(R_{i,0}, \ldots, R_{i,3})$. The source copies the packet to R_1 (shown at position $R_{1,1}$) when it comes within the transmission range. The relay R_1 copies the packet to the destination when it reaches within the communication range of the destination (shown at position $R_{1,3}$).

We note that a state space description of the evolution of the DTN system is complex; the state description contains the identities of packets in the source queue as well as the number of packet copies that are yet to be copied to the relays in addition to the identities of the packets that are carried by all the relays. This state evolves according to the spray and wait protocol described above, with the state changing at points of packet arrivals, source-relay meetings, and relay-destination meetings.

Our objective is to obtain analytical insights into the stability of the source queue as well as the different types of delays experienced by the packets arriving to the source. We denote

*This assumption is valid if we have large number of relay nodes, since the ratio of number of source-relay meetings to the number of source-destination meetings is $O(\frac{1}{N})$ which is very small.

†We note that when there is no feedback about packet delivery the source may copy a packet to a relay which has already delivered a previous copy of that packet.

the number of packets queued up at the source buffer at time t as $Q(t)$. The average queue length is then defined as

$$\overline{q_q} = \limsup_{\tau\to\infty} \frac{1}{\tau}\int_{t=0}^{\tau} \mathbb{E}Q(t)dt,$$

where the expectation is with respect to the randomness of arrivals, intermeeting times, and choice of packets when copying. We say that the source queue is stable if $\overline{q}_q < \infty$. We define the stability threshold λ^* as

$$\lambda^* = \sup\{\lambda : \overline{q_q} < \infty\}.$$

The stability region is the set of arrival rates $[0, \lambda^*)$.

We note that the i^{th} packet experiences a waiting delay from its time of arrival till the first copy is made to a susceptible relay. In the no-feedback case, we note that the i^{th} packet stays in the source queue until K copies have been made. We define the queueing delay $D_{q,i}$ experienced by the i^{th} packet as the total waiting time at the source node until the packet has been copied to K susceptible relays. In the instant-feedback case, the queueing delay is the minimum of the time from packet arrival to either the packet delivery or the time till K copies have been made. We define the delivery delay $D_{d,i}$ as the time from when the first copy is made to the earliest time when any relay infected with a copy of this packet meets the destination. An illustration of $D_{q,i}$ and $D_{d,i}$ is shown in Figure 2. The performance metrics that we are interested in are: (a) the average queueing delay $\overline{d_q}$, and (b) the average delivery delay $\overline{d_d}$ which are defined as the packet averages of $D_{q,i}$ and $D_{d,i}$ respectively over all packets. In the following sections,

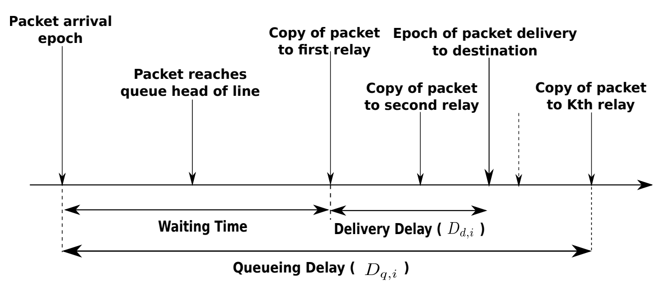

Fig. 2: Illustration of the queueing delay, and delivery delay experienced by a packet in our system model for the no-feedback case. Other important epochs during the time that the packet stays in the system are also illustrated. We note that it is possible for the packet delivery epoch to exceed the epoch at which the K^{th} copy is made.

we obtain the stability threshold λ^* for the no-feedback and instant-feedback cases. Then, for the no-feedback case, we obtain an analytical lower bound for $\overline{d_q}$ and a lower bound for $\overline{d_d}$.

III. Stability Threshold for No-Feedback

In this section we characterize λ^* for the no-feedback system. We first obtain an upper bound on λ^* by considering a *saturated queue* system, i.e., a system with a source queue which always has packets. We note that the average number of packet copies made from the source queue or average number of packet copies delivered to the destination under the saturated queue assumption is an upper bound to $K\lambda^*$, since this is the maximum rate of service or maximum throughput of packet copies. We note that under the saturated queue assumption each relay's buffer length process $B^r(t)$ evolves independently according to a CTMC with transition rates as in Figure 3. The embedded Markov chain (EMC) corresponding to the above CTMC has transition probabilities which are also shown in Figure 3. We note that the EMC is periodic. Suppose

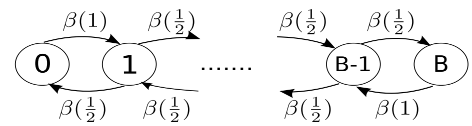

Fig. 3: The transition diagram for the CTMC and it's EMC for the $B^r(t)$ process under saturated queue assumption for no-feedback. The transition rates for the CTMC are shown along with the transition probabilities (in parenthesis) for the EMC.

$\widehat{\pi}(0), \widehat{\pi}(1), \ldots, \widehat{\pi}(B)$ denote the stationary distribution of the EMC. Then it can be shown that

$$\widehat{\pi}(0) = \widehat{\pi}(B) = \frac{1}{2B}, \text{ and } \widehat{\pi}(b) = \frac{1}{B}, \forall i \in \{1, \ldots, B-1\}.$$

For the EMC (or a $B^r(t)$), we note that whenever a downtransition occurs there is a packet copy delivery from relay r to the destination (or whenever an uptransition occurs there is a packet copy from the source to relay r). Assuming that each downtransition leads to a unit reward, the average number of downtransitions per time or the average number of packet copies per time is given by the time average reward associated with the EMC. The time average reward associated with the EMC can be obtained using Markov renewal reward theorem [17, Section D.3.3] as

$$\frac{\widehat{\pi}(B) + \sum_{b=1}^{B-1} \frac{\widehat{\pi}(b)}{2}}{\frac{\widehat{\pi}(0)}{\beta} + \frac{\widehat{\pi}(B)}{\beta} + \sum_{b=1}^{B-1} \frac{\widehat{\pi}(b)}{2\beta}} = \frac{B}{B+1}\beta.$$

We note that the factor $\frac{1}{2}$ multiplies $\widehat{\pi}(b)$ in the numerator since only one out of the two transitions contributes to the reward. We note that since there are N relays whose $B^r(t)$ processes evolve independently of each other, an upper bound on the packet copy throughput is given by $\frac{B}{B+1}\beta N$. Since this is an upper bound on the saturation throughput for packet copies, we have that $\lambda^* \leq \frac{B}{B+1}\frac{\beta N}{K}$. We now show that this upper bound is achievable arbitrarily closely by showing that the average queue length $\overline{q_q} < \infty$ for $\lambda < \frac{B}{B+1}\frac{\beta N}{K}$. We have the following result

Theorem III.1. *The stability region for the no-feedback system is* $\left\{\lambda : \lambda < \lambda^* = \frac{B}{B+1}\frac{\beta N}{K}\right\}$, *where* λ^* *is the stability threshold.*

The proof is given in Appendix C. We note that for $B = 1$ we recover the stability region result obtained in our previous work [7][‡]. For $B = 1$ we have that the stability threshold is $\frac{\beta N}{2K}$. As $B \to \infty$ we have that the stability threshold is

[‡]We note that recurrence of an underlying CTMC was shown in our previous work [7] rather than finiteness of the average queue length.

$\frac{\beta N}{K}$ which is twice what we have for $B = 1$; thus the stability threshold only doubles when the buffer size increases from one to infinity. In fact, in order to achieve 95% of the maximum value of stability threshold, a buffer size of only 19 is required. We note that Lee et al. [8]showed that the stability threshold is $\Theta(N\beta)$ for $B = \infty$ while we obtain that it is exactly $N\beta$ for $K = 1$. Using a pooled-buffer assumption [8] also showed that the stability threshold is $\Theta\left(\frac{B}{B+\rho} N\beta\rho\right)$ (for a constant load factor ρ) while we show that it is exactly $\frac{B}{B+1} N\beta$ for $K = 1$.

IV. Stability properties for instant-feedback

In this section we discuss the stability properties of the instant-feedback DTN. In this case we have results only for the case where the relays have $B = 1$, or are extremely constrained and are able to carry only one packet copy at a time.

We first obtain an upper bound on the stability threshold by using a saturated queue assumption as in the case of the system without feedback. Under the saturated queue assumption, we have that the average rate of packet copies or the saturation throughput of packet copies can be obtained from a CTMC $N_r(t)$, where $N_r(t)$ is the number of relay nodes which are carrying a packet at time t. The transition diagram of $N_r(t)$ is shown in Figure 4. We note that the left to right transitions at

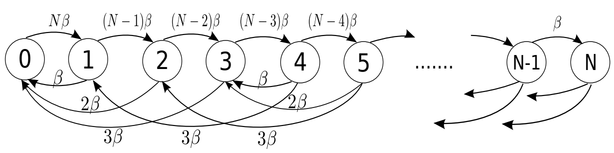

Fig. 4: The transition diagram for the $N_r(t)$ CTMC for $K = 3$ under saturated queue assumption for instant-feedback.

rate $n\beta$ in state n are due to any $N-n$ relays without a packet copy (that is a free buffer space since $B = 1$) meeting the source and obtaining a packet copy (there is always a packet to be copied since the queue is assumed to be saturated). An important feature of the $N_r(t)$ process is that $N_r(t)$ is also the number of packet copies which are carried by the relays at a time t (since $B = 1$). Suppose $N_r(t) = n$ and $n = lK + m$, then we can infer that there are m copies of a packet and K copies of l different packets in transit. Now since there is instantaneous feedback, when a relay meets the destination, the $N_r(t)$ process has a right to left transition of two types. Again with $N_r(t) = n$ we observe that with rate $m\beta$, any relay which is carrying one of the m copies of a packet will meet the destination and then there is a right to left transition of m steps. Also with rate $lK\beta$ any of the relays that are carrying one of the K copies of one of the l packets will meet the destination and then there is a right to left transition of K steps. This is illustrated in Figure 4 for $K = 3$.

Suppose we associate a reward of 1 unit with every down-transition in the $N_r(t)$ process. Then the time average reward is average saturation throughput of packets out of the saturated source queue and is our upper bound on the stability threshold. We obtain this time average saturation throughput numerically by solving for the stationary distribution of the EMC associated with $N_r(t)$, associating a reward with downtransitions, and using Markov renewal reward theorem as in the no-feedback case. However, a closed form expression is yet to be obtained for the upper bound in this case. We compare the throughput obtained for instant-feedback systems to no-feedback for the same N, β, and K values in Figure 5. Naturally the saturation throughput for systems with instant-feedback is more than for no-feedback. Although the plots suggest that the saturation throughput with instant-feedback is a constant multiple of that with no-feedback, it is not so; there is still a residual variation with K. We note that even

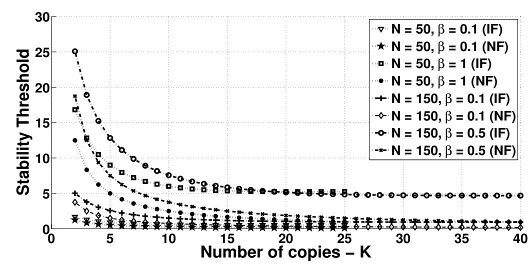

Fig. 5: Comparison of the saturation throughput (stability threshold) for instant-feedback systems (IF) and no-feedback (NF) systems

though the upper bound is not obtained analytically, it can be shown that the upper bound is achievable arbitrarily closely using a multislot Lyapunov drift proof as in the no-feedback case. Intuitively, this is because once the source queue length is large, then the average service rate out of the source is described by $N^r(t)$ as in the no-feedback case. Because of space constraints, here we state without proof that any packet arrival rate which is strictly less than the above saturation throughput can be stabilized.

V. Delay analysis for no-feedback

A. Queueing delay

For systems with no-feedback we obtain a lower bound on the queueing delay of packets at the source queue by considering another system with the same arrival rate λ but with infinite relay buffer size.

We consider the arrival process of packet copies rather than packets to the source queue. From the system model discussed in Section II we have that the source queue has a batch Poisson arrival process of packet copies with rate λ and batch size of K. The packet copy arrival rate is λK. From Theorem III.1 we have that $\lambda^* = \frac{N\beta}{K}$ when $B \to \infty$. We note that when the buffer size is infinite and the source queue is saturated, each time a relay meets a source, there is a packet copy to the relay[§]. Suppose we consider a scenario where the arrival rate $\lambda \approx \frac{N\beta}{K}$. Then we can assume that almost always there is a packet copy whenever a relay meets a source. Then the service time seen by a packet copy is exponentially distributed with rate $N\beta$,

[§]Since the source queue is saturated, it is possible to find a packet copy which is not being currently carried by the relay.

and each packet copy's service time is independent of another. Thus the source queueing process can be approximated for λ close to $\frac{N\beta}{K}$ by a $M^{[K]}/M/1$ system; i.e., a $M/M/1$ queue with batch arrivals of size K and exponential service times of rate $N\beta$. The average queue length of packet copies is then [18, Chapter 2, Section 2.10.1]

$$\overline{q_q} = \frac{\rho(K+1)}{2(1-\rho)}, \rho = \frac{\lambda K}{N\beta}.$$

We note that the service of packet copies is not in first in first out (FIFO) order in our system. However, since the above formula has been derived by considering the stationary distribution of a Markov chain where the state of the chain is the number of packet copies in the system, the order of service does not matter. An approximation to the average queue length of packets is then $\overline{q_q}/K$. The average queueing delay $\overline{d_q}$ of packets is then $\overline{q_q}/K\lambda$ using Little's law.

We note that the above approximation for the queueing delay could be applied to study the queueing delay performance of buffered systems where B is large. For extremely constrained relays (i.e. $B = 1$) we have presented delay analysis in [7].

In Figures 6 and 7 we compare the average queueing delay, obtained using simulations, for systems with $B = 1, 5, 10, 100,$ and $B = 500$ with the approximation that we have obtained above. We observe that the approximate average queueing delay for $B = \infty$ is a lower bound to the average queue lengths for finite B for the same λ. We also compare

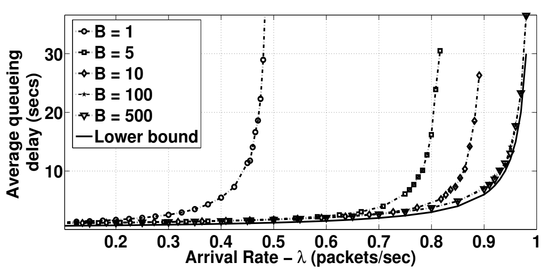

Fig. 6: The average queueing delay experienced by the packets at the source for a DTN with $N = 50$, $K = 5$, and $\beta = 0.1$. The average delay for maximum buffer sizes $B = 1, 5, 10, 100,$ and 500 are compared with the analytical average queue length (lower bound) obtained for infinite buffer sizes. The stability threshold is 1 for the infinite buffer case.

the average queueing delay, obtained using simulations, for systems with $B = 100, K = 5, \beta = 0.1$ and arrival rates of 0.8 and 0.85 with the approximation as a function of the number of relay nodes N in Figure 8. We observe that the analytical lower bound is close to the simulated values. We compare the average queueing delay, obtained using simulations, for systems with $B = 100, N = 50$ and 100, $\beta = 0.1$ and arrival rate of 0.7 with the analytical lower bound as a function of the number of copies K in Figure 9. The deviation of the average queueing delay from the lower bound for larger K can be explained as follows. For larger K for a fixed N, in the actual system the nodes would have to wait to meet a relay

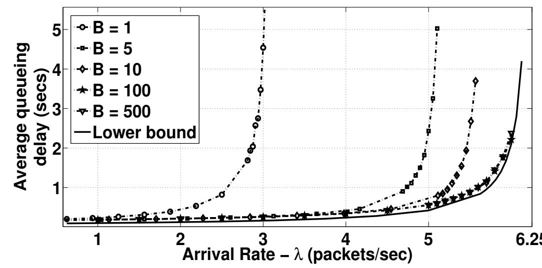

Fig. 7: The average queueing delay experienced by the packets at the source for a DTN with $N = 250$, $K = 20$, and $\beta = 0.5$. The average delay for maximum buffer sizes of $B = 1, 5, 10, 100,$ and 500 are compared with the analytical average queue length (lower bound) obtained for infinite buffer sizes. The stability threshold is 6.25 for the infinite buffer case.

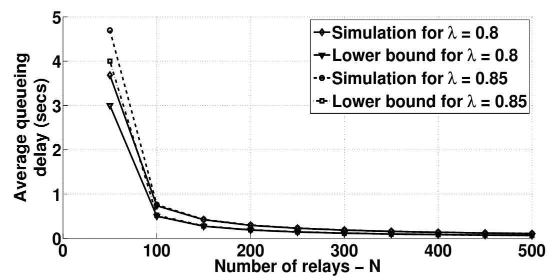

Fig. 8: The average queueing delay experienced by the packets at the source as a function of the number of relay nodes N, for a DTN with $B = 100$, $K = 5$, $\beta = 0.1$, and arrival rates of 0.8 and 0.85.

which is not carrying a duplicate copy of a packet. However, the lower bound does not take care of this particular feature of the actual system.

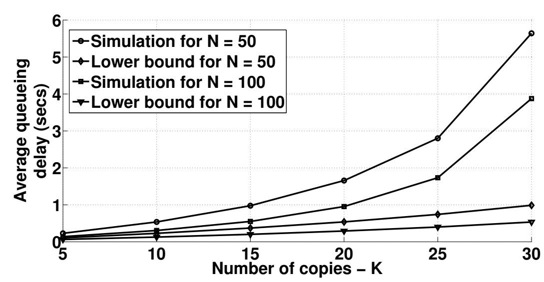

Fig. 9: The average queueing delay experienced by the packets at the source as a function of the number of copies K, for a DTN with $B = 100$, $\lambda = 0.7$, $\beta = 0.5$, and number of nodes N being 50 and 100.

B. *Delivery delay*

Another component of the delay is the average delivery delay, which is the expected time taken from the first source to relay packet copy till the first relay to destination packet delivery. A lower bound on the average delivery delay can be obtained using the CTMC shown in Figure 10. The CTMC's state is the number of unique relays to which copies of a single packet has been made. The CTMC models the evolution of this number in a network with infinite buffers and under the assumption that there is only one packet in the network at a time (lone packet assumption). The average delivery delay for that lone packet is the average time taken for the CTMC starting in state 1 to hit the special terminating state ϕ. When the number of relays is j, the maximum rate at which the packet will be delivered will be $j\beta$ since there are j relays which can meet the destination to deliver a packet copy. The use of this maximum rate is one of the reasons why we obtain a lower bound on the delivery delay in our original system. The actual delivery would happen only if at a meeting this particular packet is chosen amongst all the packets to be copied to the destination; under the lone packet assumption this is true. We also note that when the number of relays is j there are $(N-j)$ other relays which the source can meet to make a new copy of the packet (since all relays are assumed to have infinite buffer space, such a copy can happen). Suppose $t(j)$

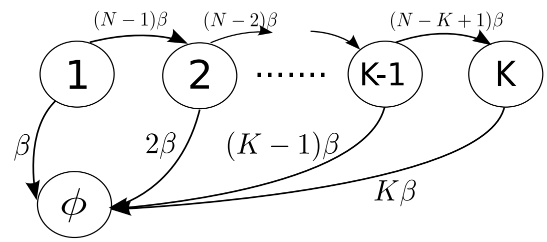

Fig. 10: The transition diagram for the CTMC modelling the number of unique relays to which a packet has been copied. The expected time to hit state ϕ starting from state 1 is the expected delivery delay of the packet under the lone packet assumption.

represents the expected time to hit ϕ starting from j. We have that $t(1)$ is the expected delivery delay. The following set of equations can be obtained by considering the transitions of the CTMC in Figure 10.

$$t(j) \quad = \quad \frac{1}{N\beta} + \frac{N-j}{N} t(j+1),$$

for all $j \in \{1, \ldots, K-1\}$. We also note that

$$t(K) \quad = \quad \frac{1}{K\beta}.$$

We note that we spend on average $\frac{1}{N\beta}$ time in every state $j \in \{1, \ldots, K-1\}$. Then, we directly hit ϕ with probability j/N from j, or with probability $(N-j)/N$ we go to $j+1$ from which the expected time to hit ϕ is $t(j+1)$.

For the system under consideration, the average delivery delay would differ from the lower bound due to two reasons: (a) the rate at which more copies of the packet is made, once the first copy has been made would be smaller than assumed since the relays have finite buffers and would be carrying other packet copies, and (b) the rate at which a copy would be delivered to the destination when a relay meets the destination would be less since a particular packet is only one amongst the packets that are currently carried by the relay, so the delivery of a particular packet happens with probability less than one at a relay-destination meeting. We compare the simulated delivery delay with the lower bound as a function of λ in Figure 11. We observe that as stated before, the delivery delay for the system is close to the lower bound at small arrival rates (since the bound assumes a lone packet). We note that the average delivery delay does not grow unbounded as a function of λ (it is always bounded above by $\frac{1}{\beta}$ since this is the average delay with just one relay). As λ increases (consider $B = 1$) we observe that the delivery delay approaches its maximum value. We compare the simulated delivery delay

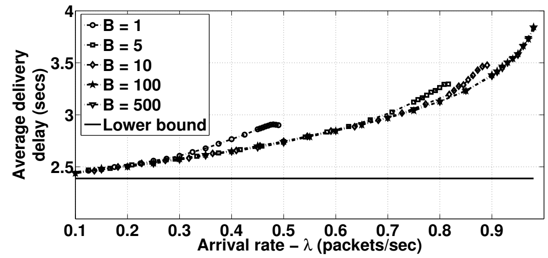

Fig. 11: The average delivery delay experienced by the packets at the source for a DTN with $N = 50$, $K = 5$, and $\beta = 0.1$. The average delay for maximum buffer sizes $B = 1, 5, 10, 100$, and 500 are compared with the lower bound under the lone packet assumption.

with the lower bound as a function of N in Figure 12. We find that the simulated values and the lower bound coincide. For large N and $B = 100$ we expect that the assumptions made in our derivation of the bound hold. In Figure 13 we

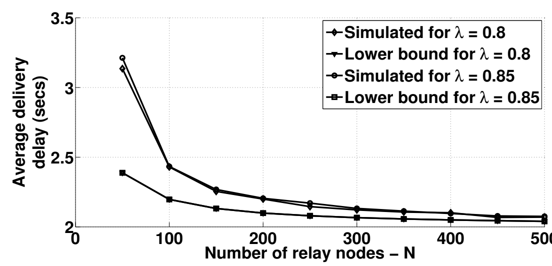

Fig. 12: The average delivery delay experienced by the packets at the source as a function of the number of relay nodes N. We consider a DTN with $B = 100$, $K = 5$, $\beta = 0.1$, and arrival rates of 0.8 and 0.85.

compare the simulated delivery delay with the lower bound as a function of K for a fixed N. For a fixed N as K increases, the source might not meet relays which are not carrying a duplicate copy of the packet at the transitions rates assumed in the CTMC shown in Figure 10. We observe that as K increases the simulated delivery delay deviates from the lower bound.

VI. Conclusions

In this paper, we considered the stability properties of a single source single destination DTN with multiple relay nodes employing two hop relaying. We considered the effect of the buffer size employed at the relay on the stability threshold and obtained that the stability threshold is at most doubled for a DTN with infinite buffer relays as compared to a DTN with

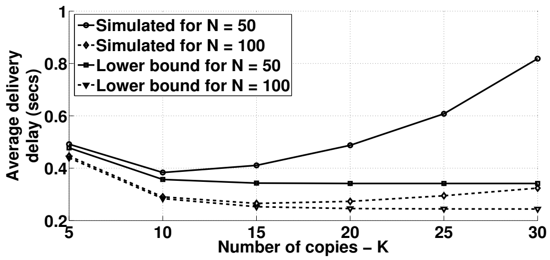

Fig. 13: The average delivery delay experienced by the packets at the source as a function of the number of copies K. We consider a DTN with $B = 100$, $\lambda = 0.7$, $\beta = 0.5$, and number of nodes N being 50 and 100.

unit buffer relays. We employed a multi-slot Lyapunov drift criterion for obtaining the stability threshold. We also studied the average queueing delay and average delivery delay in systems with buffered relays and found that simple analytical approximations for the average queueing delay can be obtained for the case where the buffer size is large (compared to our previous work [7] where we considered unit buffer size). We also obtained a lower bound on the average delivery delay, which is observed to be close to the actual when the number of nodes N is large.

References

[1] Amin Vahdat, David Becker, et al. Epidemic routing for partially connected ad hoc networks. Technical report, Technical Report CS-200006, Duke University, 2000.

[2] Thrasyvoulos Spyropoulos, Konstantinos Psounis, and Cauligi S Raghavendra. Spray and Wait: an efficient routing scheme for intermittently connected mobile networks. In *Proceedings of the 2005 ACM SIGCOMM workshop on Delay-tolerant networking*, pages 252–259. ACM, 2005.

[3] Thrasyvoulos Spyropoulos, Konstantinos Psounis, and Cauligi S Raghavendra. Spray and focus: Efficient mobility-assisted routing for heterogeneous and correlated mobility. In *Fifth Annual IEEE International Conference on Pervasive Computing and Communications Workshops.*, pages 79–85. IEEE, 2007.

[4] John Burgess, Brian Gallagher, David Jensen, and Brian Neil Levine. Maxprop: Routing for vehicle-based disruption-tolerant networks. In *INFOCOM*, volume 6, pages 1–11, 2006.

[5] Aruna Balasubramanian, Brian Levine, and Arun Venkataramani. DTN routing as a resource allocation problem. *ACM SIGCOMM Computer Communication Review*, 37(4):373–384, 2007.

[6] Jingfeng Xue, Jiansheng Li, Yuanda Cao, and Ji Fang. Advanced PROPHET routing in delay tolerant network. In *International Conference on Communication Software and Networks.*, pages 411–413. IEEE, 2009.

[7] S. Yasodharan, B. S. Vineeth, and C. Singh. Stability and delay analysis of delay tolerant networks with random packet arrivals. In *International Conference on Communication Systems and Networks (COMSNETS)*, pages 367–374, Jan 2017.

[8] Uichin Lee, Kang-Won Lee, Soon Young Oh, and Mario Gerla. Understanding the capacity and delay scaling laws of delay tolerant networks: A unified approach. *UCLA Technical Report: TR-070020*, 2008.

[9] J. D. Herdtner and E. K. P. Chong. Throughput-storage tradeoff in ad hoc networks. In *Proceedings of 24th Annual Joint Conference of the IEEE Computer and Communications Societies.*, volume 4, pages 2536–2542, March 2005.

[10] Veeramani Mahendran and C Siva Ram Murthy. Buffer dimensioning of DTN replication-based routing nodes. *IEEE Communications Letters*, 17(1):123–126, 2013.

[11] Ari Keränen, Jörg Ott, and Teemu Kärkkäinen. The ONE simulator for DTN protocol evaluation. In *Proceedings of the 2nd international conference on simulation tools and techniques*, page 55, 2009.

[12] Venkatesh Ramaiyan, Eitan Altman, and Anurag Kumar. Delay optimal scheduling in a two-hop vehicular relay network. *Mob. Netw. Appl.*, 15(1):97–111, February 2010.

[13] Robin Groenevelt, Philippe Nain, and Ger Koole. The packet delay in mobile ad-hoc networks. *Performance Evaluation*, 62(1–4):210 – 228, October 2005.

[14] Xiaolan Zhang, Giovanni Neglia, Jim Kurose, and Don Towsley. Performance modeling of epidemic routing. *Computer Networks*, 51(10):2867–2891, July 2007.

[15] Eitan Altman, Tamer Basar, and Francesco De Pellegrini. Optimal monotone forwarding policies in delay tolerant mobile ad-hoc networks. *Performance Evaluation*, 67(4):299–317, April 2010.

[16] C. K. Singh, Eitan Altman, A. Kumar, and R. Sundaresan. Optimal forwarding in delay tolerant networks with multiple destinations. *IEEE/ACM Transactions on Networking*, 21:1812–1826, 2013.

[17] Anurag Kumar, D Manjunath, and Joy Kuri. *Wireless networking*. Morgan Kaufmann, 2008.

[18] Sanjay K Bose. *An introduction to queueing systems*. Springer Science & Business Media, 2013.

[19] B. S. Vineeth and C. Singh. Stability properties of delay tolerant networks with buffered relay nodes. *https://vineethbs.github.io/papers/DTN-Buffer-Extended.pdf*, 2017.

[20] Robert G Gallager. *Stochastic processes: theory for applications*. Cambridge University Press, 2013.

Appendix A
A stochastically larger system

In this appendix, we show a stochastic dominance result that is useful for the proof of Theorem III.1. The stochastic dominance result is obtained by considering a modified Spray and Wait (SW) protocol that operates as follows. The modified SW protocol copies packets to relays at relay source meeting times only if the total number of packet copies in the source queue is strictly greater than KB[¶]. If the total number of packet copies is less than or equal to KB then no packets are copied. We denote the system with this modified protocol as the (D) system, while the original system is denoted as the (R) system. For the (D) system we also assume that when a packet arrival occurs so that the total number of packet copies in the source queue increases above KB then all relay buffers are instantaneously filled with copies of a dummy packet, all of which have to be delivered to the destination at relay destination meeting times. The delivered dummy packets are discarded by the destination. We now show that the queue length of packet copies for the (D) system is always greater than or equal to that in the (R) system.

Lemma A.1. *Let $Q^D(t)$ and $Q^R(t)$ be the number of packet copies in the source queue for the (D) system and the (R) system respectively. Then under the same initial system conditions, sample path of arrivals, source-relay meetings, and relay-destination meetings we have that $Q^D(t) \geq Q^R(t)$, for all $t \geq 0$.*

Proof. We consider the evolution of both (D) and (R) systems under the same sample path of arrivals, source-relay meetings, and relay-destination meetings. We let $A(t)$ denote the cumulative number of packet copy arrivals in both the (D) and (R) system upto time t. Also let $S^R(t)$ and $S^D(t)$ denote the cumulative number of packet copies made to the relay, or

[¶]We choose KB arbitrarily; the smallest such constant for the following lemma to hold is $K(B-1)+1$.

equivalently the cumulative number of packet copies removed from the queue for the (D) and (R) system respectively upto time t.

Suppose at $t = 0$ the system starts with the same q_0 packet copies and same relay buffer occupancies for both (D) and (R) systems. Suppose $q_0 < KB$. Until $Q^D(t)$ reaches KB, there are no packet copies being made in the (D) system. But some packet copies are made in the (R) system and those copies would be removed from the source queue in the (R) system. We note that until $Q^D(t)$ reaches KB, the relay buffer evolution of (D) and (R) evolve in separate ways, with the (R) system's relay buffers carrying newer packet copies too; packets in the (D) system which are copied to the destination are also copied in the (R) system. We note that with $q_0 < KB$, there is a time when $Q^D(t)$ reaches KB from below, since there are arrivals but there is not service in the (D) system). During this time $S^R(t) \geq S^D(t)$. Since $Q^D(t) = A(t) - S^D(t)$ and $Q^R(t) = A(t) - S^R(t)$ we therefore have that $Q^D(t) \geq Q^R(t)$.

Now suppose $q_0 > KB$. Then we note that both (D) system and (R) system will evolve in the same way till $Q^D(t) = KB$. If $Q^D(t)$ never reaches KB then we have that $Q^D(t)$ is always the same as $Q^R(t)$ and we are done. Now consider the case where $Q^D(t) = KB$. At this time $Q^R(t) \leq KB$. Then we note that until the first packet arrives, there is no service from the (D) system. As in the case discussed before (where we start with $q_0 < KB$) there will be service in the (R) system so that until the first arrival after $Q^D(t) = KB$ we have that $S^R(t) \geq S^D(t)$ and therefore $Q^D(t) \geq Q^R(t)$. Now suppose a packet arrival occurs. In the (D) system all relay buffers are filled with dummy packets. Consider the evolution of the (D) and (R) systems till the time $Q^D(t)$ hits KB again. Now, we note that packet copies from the source queue can happen in both (D) system and (R) system. However, whenever a packet copy happens in the (D) system there should be a packet copy opportunity in the (R) system since the sample path of relay-source meetings are the same in both, and the number of free buffer spaces in the (D) system is always less than or equal to the number of free buffer spaces in the (R) system for every relay. The packet copy opportunity is not used by the (R) system due to two reasons: (a) there are no packets in the source queue, in which case $Q^R(t) = 0$ or (b) there are packet copies in the source queue but none can be copied to the relay since the relay is already carrying a duplicate copy. If (b) happens then we note that $Q^R(t) < KB$, since if the number of packet copies is greater than or equal to KB then there are at least B packets in the source queue and a relay with a free buffer space can at most have $B-1$ unique packet copies. We note that if such a packet copy opportunity is not used by the (R) system then until such a time, we have $S^R(t) \geq S^D(t)$ and therefore $Q^D(t) \geq Q^R(t)$. If such a non-usage occurs, then at that time instant we have $Q^D(t) > KB$ and $Q^R(t) < KB$. Starting from that time instant till the next time instant where such a non-usage occurs or when $Q^D(t) = KB$, we repeat the above argument. Therefore, we obtain that $Q^D(t) \geq Q^R(t)$ for all $t \geq 0$. □

APPENDIX B
LYAPUNOV DRIFT FOR FINITE AVERAGE QUEUE LENGTH

Suppose $X(t)$ is a CTMC with positive and bounded transition rates out of any state. Let X_n be the EMC of $X(t)$. We consider the evolution of the EMC over frames of size T slots each (where T is deterministic). We note that if $\mathcal{P}$ is the transition probability matrix of X_n then the process considered at the frame start times, $(X_0, X_T, X_{2T}, \dots)$ is also a Markov chain with transition probability matrix $\mathcal{P}^T$. We have the following lemma.

Lemma B.1. *Suppose $L(x)$ is a Lyanpunov function. If there exists a positive ϵ, a constant $0 < K < \infty$, and a constant $0 < B < \infty$ such that for every state x*

(a) $\mathbb{E}\left[L(X_{(m+1)T}) - L(X_{mT})|X_{mT} = x\right] \leq -\epsilon x + B,$

(b) $\mathbb{E}X_{mT+l} \leq \mathbb{E}X_{mT} + Kl, \forall l \in \{1, \dots, T-1\}$ *and*

in every state x for the CTMC there is a possible transition out of that state with a rate λ that is independent of x, then

$$\limsup_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \mathbb{E}X(t)dt \quad < \quad \infty.$$

Proof. We note that if there exists a positive ϵ and a positive finite B such that for every state x

$$\mathbb{E}\left[L(X_{(m+1)T}) - L(X_{mT})|X_{mT} = x\right] \leq -\epsilon x + B,$$

then it follows from [17, Section 8.2.1] that

$$\limsup_{M\to\infty} \frac{1}{M}\sum_{m=0}^{M-1} \mathbb{E}X_{mT} \quad < \quad \infty.$$

We now consider $\limsup_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \mathbb{E}X(t)dt$. We let $M = \lfloor \frac{\tau}{T} \rfloor$. We note that $\frac{1}{\tau} \leq \frac{1}{MT}$. Therefore

$$\frac{1}{\tau}\int_0^\tau \mathbb{E}X(t)dt \leq \frac{1}{MT}\int_0^\tau \mathbb{E}X(t)dt.$$

We also note that

$$\int_0^\tau \mathbb{E}X(t)dt = \sum_{m=0}^{M-1}\int_{mT}^{(m+1)T} \mathbb{E}X(t)dt + \int_{MT}^\tau \mathbb{E}X(t)dt. \quad (1)$$

We note that

$$\int_{mT}^{(m+1)T} \mathbb{E}X(t)dt \leq \frac{T\mathbb{E}X_{mT}}{\lambda} + \frac{KT^2}{\lambda},$$

since the integral is bounded above by considering a process without any downtransitions and $1/\lambda$ is an upper bound on the expected time for a transition. We also note that the last term in (1) can be similarly bounded above. We then have that

$$\limsup_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \mathbb{E}X(t)dt \leq$$
$$\limsup_{M\to\infty} \frac{1}{MT}\sum_{m=0}^{M}\left[\frac{T\mathbb{E}X_{mT}}{\lambda} + \frac{KT^2}{\lambda}\right] < \infty.$$

□

APPENDIX C
PROOF OF THEOREM III.1

We now prove that the average queue length for the source queue is finite if $\lambda < \frac{B}{B+1}\frac{N\beta}{K}$. We do this by using the stochastic dominance result from Lemma A.1. We recall the definitions of the (D) system and the (R) system from Appendix A; the (R) system is the system under consideration using the original source Spray and Wait (SW) protocol while the (D) system uses the modified SW policy which does not copy any packets if the number of packet copies is less than KB. We prove that the average queue length for the (D) system is finite if $\lambda < \frac{B}{B+1}\frac{N\beta}{K}$. Then from Lemma A.1, it follows that $\overline{q_q} < \infty$ if $\lambda < \frac{B}{B+1}\frac{N\beta}{K}$ for the system under consideration, which is the (R) system discussed in Appendix A. For brevity, we use $Q(t)$ to denote the queue length of packet copies in the (D) system in the following.

We now note that the evolution of the number of packet copies in the source queue for the (D) system can be described by a CTMC $(\mathbf{S}(t) = (Q(t), B^1(t), B^2(t), \cdots, B^N(t))), t \geq 0)$. In Figure 14 we show the transition diagram for the CTMC for the case of $N = 1$. We note that when the number of packet copies in the source queue is more than KB then at least B packets should be there in the source queue. Then at a relay meeting time, since a relay with a free buffer space can carry at most $B-1$ packets at least one packet can be copied to the relay. We do not need fine grained information about the identities of the packet in the state description of the queue evolution in this case, which simplifies the state description compared to the (R) system. We note that for the (D) system the queue length for the (D) system is bounded below by KB once KB is hit. Therefore, the CTMC $\mathbf{S}(t)$ has some transient states for which the queue length component of the state is less than KB. For simplicity, we do not describe the transient states here. The different transitions that can occur in the evolution of the CTMC are explained in Figure 14. We note that for the (R) system when the number of packet copies is less than KB, the random process $S(t)$ would not be a CTMC. However, during the time the (R) system has a number of packet copies greater than or equal to KB, the evolution can be described using $S(t)$ (the CTMC is shown in Figure 15). We note that the steady state behaviour of the (R) process is not described by the CTMC in Figure 15 which necessitates the use of the (D) system in order to prove stability using stochastic dominance. We also note from Figures 14 and 15 that for large queue lengths the evolution of the queue length process is similar in both (D) and (R) systems.

We now consider the (D) system; the evolution of which for $N = 1$ is shown in Figure 14. We note that for $N > 1$ the state space is $(N+1)$ dimensional with the buffer state variables $B^i(t)$ being bounded but not the $Q(t)$ variable. There are similar transitions between states for the general case; for example, for large enough q, there are N transitions from a state $(q, b_1, b_2, \ldots, b_N)$ to states of the form $(q-1, b_1, b_i + 1, \ldots, b_N)$ for every i, each with rate β.

We denote the EMC, embedded at the transition epochs of the CTMC as $\mathbf{S}[n]$. Here $n \in \{0, 1, 2, \ldots\}$ indexes the discrete time instants or slots. We use the following multislot Lyapunov drift result from Lemma B.1 for the EMC in order to show that the average queue length is finite for the (D) system. We divide the evolution of the EMC into frames of length T slots each; i.e, the first frame consists of the slots $(0, \ldots, T-1)$, the second of $(T, \ldots, 2T-1)$ and so on. Suppose $L(\mathbf{s})$ is a Lyapunov function of the state $\mathbf{S}[n] = \mathbf{s}$, defined as $L(\mathbf{s}) = q^2$, where q is the queue length component of the state. We have from [19, Lemma B.1] that if there exists a T, constants $c > 0$ and $d > 0$, and it holds that for every $k \in \{0, 1, \cdots\}$

$$\mathbb{E}\left[L(Q[kT+T]) - L(Q[kT])|\mathbf{S}[kT] = \mathbf{s}\right] \leq c - dQ[kT],$$

then $\overline{q_q}$ is finite.

We consider the expected Lyapunov drift of the EMC over T slots of the EMC where T will be chosen in the following. The expected drift is:

$$\mathbb{E}\left[(Q[kT+T])^2 - (Q[kT])^2|\mathbf{S}[kT] = \mathbf{s}\right]. \qquad (2)$$

We note that for the EMC we can write an evolution equation for $Q[n]$ as follows:

$$Q[n+1] = Q[n] + A[n] - R[n],$$

where $A[n] \in \{0, K\}$ and $R[n] \in \{0, 1\}$ are arrival and service random variables. We note that $A[n]$ and $R[n]$ are dependent random variables for the EMC since the queue can have an arrival or a service but not both at a transition epoch in the CTMC. Furthermore, $A[n]$ and $R[n]$ are dependent on $Q[n]$.

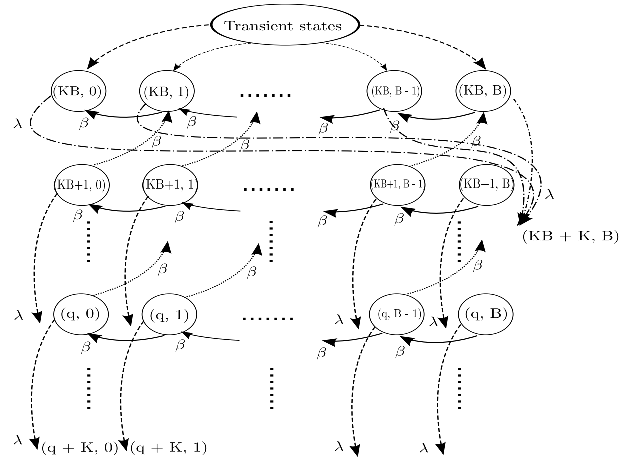

Fig. 14: The transition diagram for the CTMC $\mathbf{S}(t)$ for $N = 1$ for the (D) system; in this case $\mathbf{S}(t) = (Q(t), B^1(t))$. Downward transitions (dashed) at rate λ represent the arrival of a new packet; since the queue length is the number of copies of a packet note that the queue length increases by K. Right to left transitions occuring at rate β are due to the delivery of a packet copy at a destination. Upward (dotted) transitions at rate β are due to a packet copy from the source to the relay. There is a special transition from $(KB, -)$ to $(KB + K, B)$ in the (D) system as all the buffers are assumed to be filled once an arrival happens when the number of packet copies is KB.

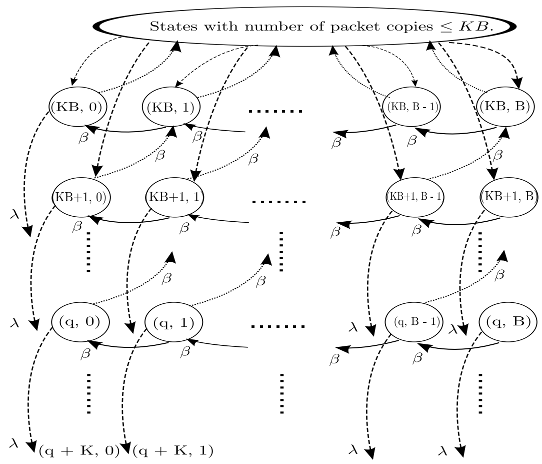

Fig. 15: The transition diagram for the CTMC $\mathbf{S}(t)$ for $N = 1$ for the (R) system; in this case $\mathbf{S}(t) = (Q(t), B^1(t))$. For the (R) system the CTMC describes the queue evolution only for the random duration of time that the number of packet copies is greater than or equal to KB. The system enters this phase of evolution from *states* where the number of packet copies is less than KB and leaves this phase of evolution from states where the number of packet copies is KB and with an upward (dotted) transitions at rate β are due to a packet copy from the source to the relay. Downward transitions (dashed) at rate λ represent the arrival of a new packet; since the queue length is the number of copies of a packet note that the queue length increases by K. Right to left transitions occuring at rate β are due to the delivery of a packet copy at a destination.

With $n = mT$, the drift in (2) can be written as

$$\mathbb{E}\left[\left(Q[n] + \sum_{m=0}^{T-1}(A[m] - R[m])\right)^2 - (Q[n])^2|\mathbf{S}[n]\right].$$

We then have that the drift is

$$2Q[n]\mathbb{E}\left[\sum_{m=0}^{T-1}(A[m] - R[m])|\mathbf{S}[n]\right] + \mathbb{E}\left[\left(\sum_{m=0}^{T-1}A[m] - R[m]\right)^2 |\mathbf{S}[n]\right],$$

which can be bounded above as

$$2TQ[n]\mathbb{E}\left[\frac{\sum_{m=0}^{T-1}(A[m] - R[m])}{T}|\mathbf{S}[n]\right] + T(K^2 + 1). \quad (3)$$

We now consider the case of $N = 1$ to show how to bound the term $\mathbb{E}\left[\sum_{m=0}^{T-1}(A[m] - R[m])|\mathbf{S}[n]\right]$. Then we will extend our method to any N. Consider a slot n at which a T slot frame starts and suppose $Q[n] = q > KB + T$. We then note that since there can at most be T services in T slots, if $q > KB+T$ then in the T-slot frame starting from n the queue length will never be KB. We denote the CTMC in Figure 14 as CTMC(A). For CTMC(A) we observe that starting from such a $q > KB+T$ any sample path of transitions for T slots can be obtained by considering another CTMC with self-transitions (or a semi-Markov process) with a transition diagram as shown in Figure 16. We will denote this CTMC as CTMC(B). We note that all transitions in CTMC(A) occur in CTMC(B). The arrival transition of rate λ in CTMC(A) is the self-transition of rate λ in CTMC(B). The other transitions in CTMC(B) are the same as that in CTMC(A). Using CTMC(B) is important since the sum $\mathbb{E}\left[\sum_{m=0}^{T-1}(A[m] - R[m])\right]$ can be characterized using the simpler CTMC(B) rather than the complete CTMC(A). For

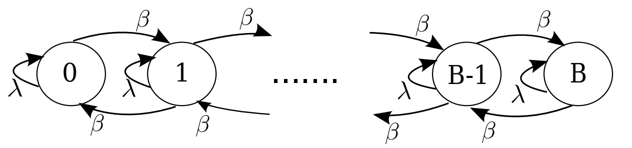

Fig. 16: CTMC(B): a simpler CTMC model used for generating sample paths with the same transitions as the CTMC(A) in Figure 14.

example, we note that $\sum_{m=0}^{T-1} A[m]$ is K times the number of downward (λ rate) transitions which happened in CTMC(A) in T transitions of the CTMC(A), which is equal to the number of self-transitions which happened in the CTMC(B) in T transitions of CTMC(B). Similarly, $\sum_{m=0}^{T-1} R[m]$ is the number of upward (β rate) transitions which happened in CTMC(A) in T transitions, which is equal to the number of left to right transitions in CTMC(B) in T transitions.

Since we are interested in $\mathbb{E}\left[\frac{\sum_{m=0}^{T-1}(A[m]-R[m])}{T}|\mathbf{S}[n]\right]$, let us try to evaluate the above expectation. We denote the EMC of CTMC(B) as EMC(B). Suppose we assume that a reward of K is associated with a self transition of EMC(B). We also assume that there is a reward of -1 associated with a left to right transition of the EMC(B). Since EMC(B) is aperiodic, irreducible, and has finite state space, we have that

$$\frac{1}{T}\mathbb{E}\left[\sum_{m=0}^{T-1}(A[m] - R[m])|\mathbf{S}[n]\right] = \mathbb{E}_{\widehat{\pi}}\left[A - R\right] + e(T),$$

where $\widehat{\pi}$ is the stationary distribution of the EMC(B), and A and R are the arrival and service random variables with the appropriate marginal joint distribution obtained from $\widehat{\pi}$, and $e(T) \downarrow 0$ as $T \uparrow \infty$. We discuss the exact form of $e(T)$ later in the proof.

We will now compute the stationary distribution $\widehat{\pi}$ of the EMC(B). We denote the stationary distribution of CTMC(B) as π. It is important to note that the stationary distribution of CTMC(B) can be obtained by considering the CTMC(B) without self loops in which case the transition diagram would be as in Figure 3. For all $b \in \{0, 1, \ldots, B\}$, we have that the stationary distribution of the CTMC(B) is $\pi(b) = \frac{1}{B+1}$. This connection to CTMCs without self loops is important when we consider $N > 1$. We emphasize that CTMC(B) without self loops is not CTMC(A).

Since $\pi(b)$ can be interpreted as the fraction of time the CTMC is in state b we have that

$$\pi(b) = \frac{\widehat{\pi}(b)}{a(b)}/\sum_{j=0}^{B}\frac{\widehat{\pi}(j)}{a(j)},$$

where $a(j)$ is the total transition rate out of state j including the self transition rate (e.g., $a(0) = \lambda + \beta, a(1) = \lambda + 2\beta$). Then, we have that

$$\widehat{\pi}(b) = \pi(b)a(b)/\sum_{j=0}^{B} \pi(j)a(j). \quad (4)$$

For EMC(B) we obtain that

$$\begin{aligned} \widehat{\pi}(0) &= \widehat{\pi}(B) = \frac{\lambda + \beta}{B(\lambda + 2\beta) + \lambda}, \text{ and} \\ \widehat{\pi}(b) &= \frac{\lambda + 2\beta}{B(\lambda + 2\beta) + \lambda}, \forall b \neq 0, B. \end{aligned}$$

We note that

$$\mathbb{E}_{\widehat{\pi}} A = \sum_b \widehat{\pi}(b) \frac{\lambda}{a(b)} K = \frac{(B+1)\lambda K}{B(\lambda + 2\beta) + \lambda}.$$

Also,

$$\mathbb{E}_{\widehat{\pi}} R = \widehat{\pi}(0) \frac{\beta}{a(0)} + \sum_{b=1}^{B-1} \widehat{\pi}(b) \frac{\beta}{a(b)} = \frac{B\beta}{B(\lambda + 2\beta) + \lambda}.$$

We note that if $(B+1)\lambda K < B\beta$ or if $\lambda < \frac{B\beta}{(B+1)K}$ then there exists an ϵ such that $\lambda + \frac{\epsilon}{(B+1)K} < \frac{B\beta}{(B+1)K}$.

We then have that $\frac{1}{T}\mathbb{E}\left[\sum_{m=0}^{T-1}(A[m] - R[m])\middle|\mathbf{S}[n]\right] =$

$$-\frac{B\beta - \epsilon - (B+1)K\lambda}{B(\lambda + 2\beta) + \lambda} + e(T) - \frac{\epsilon}{B(\lambda + 2\beta) + \lambda}. \quad (5)$$

For any ϵ since $e(T) \downarrow 0$ as $T \uparrow \infty$ we choose a T large enough so that $e(T) - \frac{\epsilon}{B(\lambda+2\beta)+\lambda} < 0$. Then, (5) is denoted as $-d$, where $d \in (0, T]$; the upper bound of T arises since the drift in T slots cannot be larger than the maximum number of services in T slots. Then, we have shown that whenever at the start of a frame of length T if the queue length is such that $Q[n] > KB + T$ then the expected Lyapunov drift over T slots is:

$$\leq -2TQ[n]d + c_1, c_1 = T(K^2 + 1)$$

Now suppose we consider those frames for which the queue length at the start of the frame is less than or equal to $KB+T$. For such frames, we note that that the expected Lyapunov drift is:

$$\begin{aligned} &\leq -2TQ[n]d + 2TQ[n]K + 2TQ[n]d + c_1, \text{ or,} \\ &\leq -2TQ[n]d + 2KT(KB + T) + 2T^2(KB + T) + c_1. \end{aligned}$$

Thus for all queue lengths at the start of a frame of length T slots, we have that the expected Lyapunov drift is

$$\leq \quad -2TQ[n]d + c,$$

where $c = 2KT(KB+T) + 2T^2(KB+T) + c_1$. Then from Lemma B.1 we have that the average queue length for the (D) system is finite. Thus, for $N = 1$ we have that if $\lambda < \frac{B\beta}{(B+1)K}$ then the (R) system has a finite average queue length, and is therefore stable.

We now consider the case of general N and also discuss the form of $e(T)$. For general N a part of the transition diagram for the CTMC for $\mathbf{S}(t)$ is shown for a $q > KB$ in Figure 17 (note that some transitions may not exist depending on whether $b_i = 0$ or B). We denote this CTMC again as CTMC(A). As in the case of $N = 1$, for the purpose of finding out $A[m]$ and $R[m]$ we can consider a simpler CTMC(B) with self loops which is also shown in Figure 18. We proceed as in the case of $N = 1$ by obtaining the stationary probability $\widehat{\pi}$ of the EMC(B) associated with CTMC(B). We recall the important fact that the stationary distribution π of CTMC(B) can be obtained by considering the CTMC(B) itself without self-loops. We then note that CTMC(B) without self-loops decomposes into N independent CTMCs each evolving according to the transition diagram in Figure 3. We denote the vector $(b_1, b_2, \ldots, b_N)$ by $\mathbf{b}$. Then we obtain that $\pi(\mathbf{b}) = \frac{1}{(B+1)^N}$. Using (4) we have that

$$\widehat{\pi}(\mathbf{b}) = \pi(\mathbf{b})a(\mathbf{b})/\sum_{\mathbf{j}} \pi(\mathbf{j})a(\mathbf{j}).$$

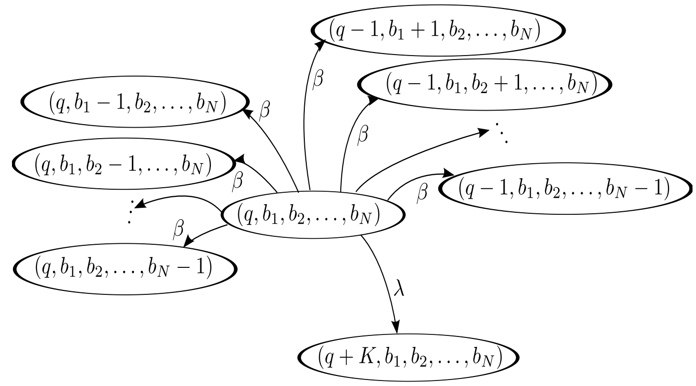

Fig. 17: A part of the CTMC transition diagram for any N

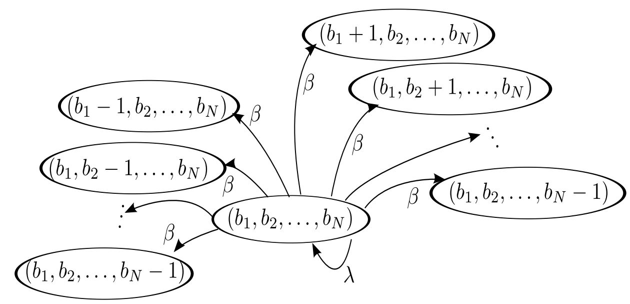

Fig. 18: A part of the CTMC(B) transition diagram for any N

Here $a(\mathbf{b})$ is the total transition rate out of state $\mathbf{b}$ which includes the self transition rate. Suppose $\mathbf{b} = (b_1, b_2, \ldots, b_N)$ is such that l of the N buffer values are zero, m of the buffer values are B. Then we have that $a(\mathbf{b}) = \lambda + \beta(l+m) + 2\beta(N - (l+m))$ since for a particular $\mathbf{b}$ every buffer state which is neither 0 or B can have an up and down transition with rate β each, and any buffer state which is either 0 or B can have an up or down transition respectively with rate β. Also, every state has a self loop of rate λ. Since $\pi(\mathbf{b}) = \frac{1}{(B+1)^N}$ we have that

$$\widehat{\pi}(\mathbf{b}) = a(\mathbf{b})/d,$$

where $d = \sum_{\mathbf{j}} a(\mathbf{j})$.

As for the case of $N = 1$, we now obtain $\mathbb{E}_{\widehat{\pi}}\left[A - R\right]$. We have that

$$\mathbb{E}_{\widehat{\pi}} A = \sum_{\mathbf{b}} \widehat{\pi}(\mathbf{b}) \frac{\lambda}{a(\mathbf{b})} K = \sum_{\mathbf{b}} \frac{\lambda K}{d}.$$

Since the total number of states is $(B+1)^N$ we have that

$$\mathbb{E}_{\widehat{\pi}} A = (B+1)^N \lambda K.$$

For a state $\mathbf{b}$ let $s(\mathbf{b})$ represent the transition rate corresponding to a left to right transition. Then we have that

$$\mathbb{E}_{\widehat{\pi}} R = \sum_{\mathbf{b}} \widehat{\pi}(\mathbf{b}) \frac{s(\mathbf{b})}{a(\mathbf{b})} = \sum_{\mathbf{b}} \frac{s(\mathbf{b})}{d}$$

We note that there are B^N states or $\mathbf{b}$-s such that $s(\mathbf{b}) = N\beta$; since every $\mathbf{b}$ which is such that no $b_j = B$ can have a left to right transition for any of the N buffer sizes. We also have that there is NB^{N-1} states which have $s(\mathbf{b}) = (N-1)\beta$ and so on. Then, it follows that

$$\begin{aligned}\sum_{\mathbf{b}} \frac{s(\mathbf{b})}{d} &= \frac{NB\beta}{d}\left[\sum_{n=0}^{N-1} \binom{N-1}{n} B^{N-1-n}\right] \\ &= \frac{NB\beta(B+1)^{N-1}}{d}.\end{aligned}$$

Therefore, we have that

$$\mathbb{E}_{\widehat{\pi}}\left[A - R\right] = \frac{K(B+1)^N}{d}\left[\lambda - \frac{B}{B+1}\frac{\beta N}{K}\right]. \qquad (6)$$

Now the drift analysis proceeds in the same way as for $N = 1$.

We now discuss the exact form of $e(T)$. We note that EMC(B) is an aperiodic, irreducible, finite state Markov chain. If p_{ij}^n is the n step transition probability of EMC(B) then we have that

$$\mathbb{E}\left[A[n] - R[n]\right] = \mathbb{E}_{p_{ij}^n}\left[A[n] - R[n]\right].$$

From [20, Theorem 4.4.2] we have that

$$p_{ij}^n \leq \widehat{\pi}(j) + poly(n)\mu^n,$$

where $poly(n)$ is a polynomial in n and μ is the second eigen value of the transition probability matrix $[p_{ij}^1]$ which is less than 1. In our case we therefore obtain that $e(T) = K' poly(T)\mu^T$ which $\downarrow 0$ as $T \uparrow \infty$ and K' is a constant.